Supporting Information for

Illuminating a Contorted Slab with a Complex Intraslab Rupture Evolution during the 2021 Mw 7.3 East Cape, New Zealand Earthquake

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Text S1. Hypocenter relocation

The regional monitoring agency, GeoNet, reported an initial hypocenter depth for the 2021 East Cape earthquake of \(\sim 80\) km (https://www.geonet.org.nz/earthquake/history/2021p169083; last accessed 06/05/2021) before later being revised to an operator-assigned default depth of \(12\) km. Global monitoring agencies also assigned fixed depths, but waveform-based depths (e.g. centroids) appear to be deeper at \(\sim 50\) km (Table S2). The global arrival-time based hypocenter estimates with fixed depths also have very large arrival time residuals (>3 s) at the closest stations located within 200 km epicentral distance, suggesting a possible incompatibility with the fixed depth solutions. Given this apparent uncertainty and mismatch in depths, we performed a relocation of the earthquake using stations at local-regional distances. We picked \(P\)- and \(S\)-wave arrival times from GeoNet stations (network code: NZ) up to \(\sim 1000\) km epicentral distance, with each arrival time assigned a weight based on picking uncertainty. The closest station is located at 100 km epicentral distance. We also included arrival times from the station NZ.RAO located to the north on the Kermadec Islands to maximize the azimuthal coverage.

For the relocation, we used the NonLinLoc software (Lomax et al., 2000, 2009), which offers robust constraints on location uncertainties compared with traditional single-event location codes, and in which we employed a travel-time dependent error, effectively giving an epicentral distance weighting. For the velocity model, we took a 1-D profile through the New Zealand Wide 3-D \(P\)- and \(S\)-wave velocity model (NZW2.2); (Eberhart-Phillips et al., 2010, 2020). We used the 1-D profile because the 2021 earthquake source region is located at the boundaries of the available 3D model, where the model defaults back to an overall 1D average, which is probably more suitable for the center of New Zealand. Beneath the area where NZW2.2 has sufficient resolution (\(\sim 35\) km), we merged the velocity model with the global AK135 model (Kennett et al., 1995). Our relocated hypocenter is as follows: time: 2021-03-04T13:27:35.71; latitude: -37.466°; longitude: 179.774°; depth 72 km, with an overall RMS residual of 0.63 s, which is acceptable given the large station distances considered. Residuals at the closest stations are small - for example at station MXZ, located at 120 km epicentral distance, the \(P\)-wave residual is 0.3 s, whereas for the USGS-NEIC solution, it is 3.3 s. The 68% confidence ellipsoid of our solution corresponds to an epicentral uncertainty of 0.03 and 0.02 degrees in longitude and latitude, respectively; the depth uncertainty is \(\pm 9\) km (see Fig. S1). We also found that this hypocenter location was robust if we instead used the AK135 global velocity model (Kennett et al., 1995).

Text S2. \(W\)-phase CMT inversion

To gain a first-order approximation of the earthquake source, we performed a moment tensor inversion using long-period regional and teleseismic waveforms (Fig. S3). We used the Grond software (Heimann et al., 2018) which uses a Bayesian bootstrapping in a probabilistic optimisation method to fully explore the solution space, and resulting uncertainties and parameter trade-offs. We used stations from the G, GT, II, and IU seismic networks located within epicentral distances of 5–90 degrees. We defined the \(W\)-phase window and frequency bandpass of 2.0–8.5 mHz based on Duputel et al. (2012). Our window includes a long-period taper on either side of the predicted \(W\)-phase window, which includes surface waves that help to better constrain source depth (e.g., Ye et al., 2017). The best-fitting source duration of 30–35 s is consistent with the source time function from the teleseismic finite fault model (Fig. 3).
The full report of the inversion result, including the waveform fits and the model
uncertainties is available at https://rokuwaki.github.io/2021EastCapeNZ/grond-report/.

Text S3. Regional centroid moment tensor (R-CMT) inversion

For R-CMT inversion, we computed Green's functions in a 1-D seismic velocity
profile from the NZW2.1 velocity model described in the section above. We used the
ISOLA software (Sokos & Zahradnik, 2008), which inverts for waveforms in the time
domain to compute moment tensors. ISOLA searches across a prescribed grid of trial-
point sources to find the deviatoric centroid moment tensor (CMT) in space and time
that maximizes the fit between the observed and synthetic waveforms. We used avail-
able strong motion and broadband waveforms from the GeoNet network (network code:
NZ; Petersen et al., 2011) located up to ~400 km epicentral distance and removed the
instrument responses. We carefully chose waveforms unaffected by clipping and non-
linear tilting due to strong ground motion from the earthquake. We found that weight-
ing the waveforms equally by epicentral distance produced the most stable results.

Low frequency, single source inversion, grid search over depth

We first inverted for a single-point source moment tensor solution using long-
period waveforms (50–100s period), searching over a grid of trial point sources be-
 tween 3 km and 139 km depth with a grid spacing of 4 km, located beneath our re-
located epicenter. The best-fitting solution gives a centroid depth of 71 km with a fault-
ing mechanism of oblique-reverse faulting, striking roughly east-west (see Fig. S4).
This mechanism is consistent with the focal mechanisms of the earthquake reported
by global monitoring agencies (USGS-NEIC, GFZ-GEOFON, IPGP-GEOSCOPE; Fig.
S10). Correlation as function of depth (Fig. S4) also shows a weaker local maxima in
waveform correlation with a normal faulting mechanism at <12 km depth, a consis-
tent feature that we discuss further below.

Higher frequency, multiple source inversion, grid search over depth

To investigate source complexity, we increased the upper frequency corner in the
inversion to 0.04 Hz (25 s) and resolved multiple sub-events using a 25 s long trian-
gular moment-rate function (Fig. S5). The first sub-event (MW 7.3) has the oblique-
thrusting mechanism found above, with a centroid time of +11 s from the origin time
(OT) and depth of 71 km. The second sub-event (MW 6.8) has a mechanism of trench-
parallel normal faulting, and has a centroid time of OT+17 s and a depth of 7 km.
Addition of the second sub-event significantly increases the total waveform variance
reduction (VR) by 24%. This normal faulting sub-event is more consistent with the
solution reported by GeoNet (Fig. S10). This configuration of deep sub-event followed
by shallow sub-event is consistent with the overall local correlation minima seen in
the low-frequency solution (Text S3).

Higher frequency, multiple source inversion, grid search a 2-D trench-parallel
oriented plane

In a final step, we solved for multiple sub-events on a trench-parallel plane of
trial point sources, similar to the teleseismic slip-rate inversion above (although with
a coarser grid spacing of 10 km along-strike and down-dip (Fig. S6). We also tested
a trench-normal plane; however this configuration produced an overall poorer fit to
waveforms. The resulting solution is similar to those above, aside from Sub-event 1 shifts 20–30 km north of the epicenter, and has a slightly shallower depth of 52 km.

**Text S4. Finite-fault potency-density inversion**

We use the flexible finite-fault inversion method developed by Shimizu et al. (2020). The method considers the uncertainty of the Green’s function by adopting the formulation of Yagi and Fukahata (2011), which explicitly introduces the error term of the Green’s function into the data covariance matrix. The method also considers the uncertainty of fault geometry by representing the fault deformation by the superposition of five-basis double couple components (Kikuchi & Kanamori, 1991), which solves a spatiotemporal distribution of potency density (Ampuero & Dahlen, 2005). The finite-fault potency inversion (Shimizu et al., 2020) adopted in this study has been proven efficient to flexibly model complex rupture evolution of large earthquakes (Okuwaki et al., 2020; Tadapansawut et al., 2021; Yamashita et al., 2021), which is suited for estimating the unknown fault geometry and the non-smooth rupture evolution related to the geometrical complexity of the fault system.

We use the vertical component of the teleseismic $P$ waveforms at 55 stations (Fig. S11). The first motion of the $P$ wave is manually picked. The data are selected to ensure azimuthal coverage and with the high signal-to-noise ratio (e.g., Okuwaki et al., 2016). The data are then deconvolved into velocity at 0.6 s sampling interval. The Green’s functions are calculated based on the method of Kikuchi and Kanamori (1991). We used the ak135 model (Kennett et al., 1995) to calculate travel time, ray parameter, and geometric spreading factors. The CRUST1.0 model (Laske et al., 2013) is used for the one-dimensional layered medium near the source region to calculate Haskel propagator in Green’s functions (Table S1). We do not apply a low-pass filter to both the observed waveforms and Green’s functions, which is intended to retrieve the detailed rupture process recorded in higher frequency components (e.g., Shimizu et al., 2020).

We use a model-plane geometry for the finite-fault model, based on the 1-week aftershock distribution (GeoNet, 2021), adopting 200° strike and 90° dip angles. The rectangular finite-fault dimension is 70-km length and 120-km width, and the griding intervals of sub-faults are 5 km × 10 km along the strike and dip directions. The slip-rate function for each source element on the model fault is represented by the linear B-splines at the temporal interval of 0.6 s. The total source duration is set as 33 s. The maximum rupture velocity is set at 5 km/s, which is fast enough to track a possible fast-propagating rupture front limited by the local shear-wave velocity near the source region (Table. S1). We use the relocated hypocenter 37.466°S, 179.774°E, and 72-km depth for the initial rupture point.

In order to evaluate a modeling sensitivity to selection of a priori model parameters, we test different model-fault geometries and the nucleation depths (Fig. S12). To evaluate a model geometry, we test the horizontal model planes with 0°-dip angle placed at different depths and a vertical model plane with 110°-strike and 90°-dip angles. To evaluate a preferred nucleation depth, we hypothesize the different nucleation depths from 7 to 107 km depths at an interval of 10 km on the optimal model geometry with the 200°-strike and 90°-dip angles. We find that the model favors a deep rupture nucleation, and the slip also favors a deeper part in slab, which lowers the variance between the observed waveforms and the synthetic waveforms calculated as
(Shimizu et al., 2021):

\[
\text{Variance} = \frac{\sum_j \sum_t (u_{j}^{\text{obs}}(t) - u_{j}^{\text{syn}}(t))^2}{\sum_j \sum_t (u_{j}^{\text{obs}}(t))^2},
\]

where \(u_{j}^{\text{obs}}(t)\) and \(u_{j}^{\text{syn}}(t)\) are the observed and synthetic waveforms at \(j\)th station. We also find that the north-south striking fault (strike/dip = 200°/90°, Variance: 0.26) better explains the observed waveforms than the east-west striking fault (strike/dip = 110°/90°, Variance: 0.28). Based on these sensitivity tests, we adopt a model-plane geometry of the 200°-strike and 90°-dip angles with the rupture nucleation point at the relocated hypo-depth of 72 km as our optimal model-plane geometry, which the entire discussions presented in this study are based on.

Resolvability of the complex source process of the 2021 East Cape earthquake has been verified by a synthetic recovery test of the real solution (Fig. S13). We use our optimal source model as an input model, and perform the same inversion procedure written above. The input model is well reproduced, including the complex fault geometry.
**Table S1.** Near-source structure used for calculating Green’s functions for the finite-fault inversion.

<table>
<thead>
<tr>
<th>$V_P$ (km/s)</th>
<th>$V_S$ (km/s)</th>
<th>Density (g/cm$^3$)</th>
<th>Thickness (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.50</td>
<td>0.00</td>
<td>1.02</td>
<td>2.64</td>
</tr>
<tr>
<td>6.00</td>
<td>3.50</td>
<td>2.72</td>
<td>1.52</td>
</tr>
<tr>
<td>6.60</td>
<td>3.80</td>
<td>2.86</td>
<td>2.28</td>
</tr>
<tr>
<td>7.10</td>
<td>3.90</td>
<td>3.05</td>
<td>3.79</td>
</tr>
<tr>
<td>8.29</td>
<td>4.59</td>
<td>3.41</td>
<td>- (below moho)</td>
</tr>
</tbody>
</table>

**Table S2.** Reported earthquake depths from monitoring agencies.

<table>
<thead>
<tr>
<th>Agency</th>
<th>Depth (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrival time based hypocentres USGS-NEIC$^1$</td>
<td>10$^*$</td>
</tr>
<tr>
<td>GFZ-GEOFON$^2$</td>
<td>35$^*$</td>
</tr>
<tr>
<td>GeoNet$^3$</td>
<td>12$^*$</td>
</tr>
<tr>
<td>Local-regional relocation (this study)</td>
<td>72</td>
</tr>
<tr>
<td>Waveform based solutions USGS-NEIC W-phase centroid$^4$</td>
<td>51</td>
</tr>
<tr>
<td>IPGP-GEOSCOPE$^5$</td>
<td>45</td>
</tr>
<tr>
<td>Global-CMT$^6$</td>
<td>52</td>
</tr>
<tr>
<td>Regional CMT (this study; 50–100 s period)</td>
<td>71</td>
</tr>
</tbody>
</table>

$^1$ U.S. Geological Survey Earthquake Hazards Program (2017)
$^2$ Bormann (2012)
$^3$ GeoNet (2021)
$^4$ U.S. Geological Survey Earthquake Hazards Program (2017); Duputel et al. (2012)
$^5$ Vallée et al. (2011); Vallée (2013)
$^6$ Dziewonski et al. (1981); Ekström et al. (2012)

*fixed depth*
Figure S1. Relocation result. Left panels show the map and cross-sections of the sample locations. The cross marker shows the relocated hypocenter. The small black dots indicate the scatter cloud samples from the hypocentre probability density function. Right panel shows the station distribution (triangle) and the relocated epicenter (star). The dashed lines are the plate boundaries (Bird, 2003).
Figure S2. Relocated 1-week aftershock distributions with the different maximum depth uncertainty. The left panel shows the map view of the relocated aftershocks. The dashed line is the trench (Bird, 2003). The middle panel shows the cross section of the relocated aftershocks within the rectangle shown in the left panel. The star shows our relocated hypocentre. The right panel shows the histogram of the event counts every 5-km bin.
Figure S3. Result of our W-phase moment tensor inversion. Left panel shows the histogram of the sample distribution in centroid depth. Right-top panel shows the centroid location (beachball) and the relocated epicenter (star). The dashed line is the trench (Bird, 2003). Right-bottom panel is an azimuthal equidistant projection of the station distribution (triangle). The star shows the centroid location. The dashed lines are the epicentral distances at 30° and 90°.
Figure S4. Results from low-frequency, single source regional centroid moment tensor inversion. The left-hand panel shows waveform correlation as a function of depth, and the corresponding double couple solution from best-fitting CMT at each depth. The horizontal gray dashed line shows the best-fitting centroid depth. The inset map shows the stations used in the inversion. The right-hand panel shows the waveform fits (50–100 s period) for the north-south (NS), east-west (EW), and vertical (Z) components. The station code is denoted on each panel.
Figure S5. Results from high-frequency, two-point-source regional centroid moment tensor inversion, searching over a line of depth trial point sources. The left-hand panel shows waveform correlation as a function of depth for both sub-events, and the corresponding double couple solution from best-fitting CMTs at each depth. The horizontal gray dashed line shows the best-fitting centroid depth in each case. The inset map shows the stations used in the inversion. The right-hand panel shows the waveform fits (25–100 s period) for the north-south (NS), east-west (EW), and vertical (Z) components. The station code is denoted on each panel.
Figure S6. Results from high-frequency, two-point-source regional centroid moment tensor inversion, searching over 2-D trench-parallel oriented plane. The top panels show waveform correlation in a 2D search area for sub-events 1 and 2. The horizontal locations of the searching grid are shown in the right-top panel with the star showing the relocated epicenter and the dashed line as a trench (Bird, 2003). The background colour shows the waveform correlation between the observed and synthetic waveforms. The contour shows the 90% of maximum correlation. The yellow beachball is the corresponding double couple solution from best-fitting CMT. The gray dashed lines show the best-fitting centroid location in each case. The right-bottom map shows the stations (triangle) used in the inversion and the relocated epicenter (star). The left-bottom panel shows the waveform fits (25–100 s period) for the north-south (NS), east-west (EW), and vertical (Z) components. The station code is denoted on each panel.
**Figure S7.** Double-couple percentage (%) for each source element of the finite-fault model. The left panel shows the cross section of the optimal finite-fault model. The beach ball shows a lower-hemisphere stereographic projection of the moment-tensor solution, which is not rotated according to the model geometry, but from top view. The right panel shows the histograms of double-couple (DC) ratio of the moment tensor solutions for the shallow and deep layers.

**Figure S8.** Centroid moment tensor solutions in the study area for the events $M_W$ 5 or larger before 11th March 2021 from GeoNet (2021), the GCMT project (Dziewonski et al., 1981; Ekström et al., 2012), and the USGS NEIC (U.S. Geological Survey Earthquake Hazards Program, 2017). The yellow beachball is the 2021 East Cape earthquake. Grey and pink beach balls indicate events from before and after the 2021 East Cape earthquake, respectively. Red beachball is the solution for the $M_W$ 7.1 2001-08-21 earthquake (not registered in GeoNet (2021) catalog). The dashed line gives the approximate location of the subduction trench (e.g., Bassett et al., 2010). The star marks our relocated epicenter.
Figure S9. The comparison between the P-axis azimuth distribution from our finite fault model (Fig. 2) and the Slab2 model (Hayes, 2018; Hayes et al., 2018).

Figure S11. Waveform fitting between the observed (black) and the synthetic waveforms (red) for the optimal finite-fault model with the station code, azimuth, and epicentral distance from the epicenter. The inset is the azimuthal equidistant projection of the station distribution (triangle). The star shows the epicenter, and the dashed lines are the epicentral distances at 30° and 90°.
Figure S12. Summary of sensitivity tests for the finite-fault modeling. The left panels show the model-plane geometries and the initial rupture point (star). The yellow panel and the yellow star correspond to the optimal model in each case. The right panels show the variance (waveform fits) distributions. $\phi_m$ and $\delta_m$ represent the : model strike and model dip angles.
**Figure S13.** Synthetic test for the finite-fault inversion. The top panels show the input model from the optimal finite-fault model (Fig. 2). The bottom panels show the inverted solution using the synthetic waveforms forward-calculated from the input model. The inversion procedure is the same as used for obtaining the optimal model, described in Text S4.
Figure S14. Comparison of the finite-fault solutions obtained by this study and by the conventional inversion scheme. “Restricted modeling” solved only the two components of slip vectors (rake angle and slip) along the model-domain geometry and did not allow the variations of focal mechanism during the rupture. The middle panel shows the waveform fitting at the selected stations shown in the right panel.
Figure S15. Spatiotemporal distribution of our preferred finite-fault solution. The result is projected along a direction of the model strike (110° azimuth). The contours show the slip rate distributions. The black lines show the reference rupture speeds.
Figure S16. Seismicity around the source region of the 2021 East Cape earthquake from GeoNet (2021) and U.S. Geological Survey Earthquake Hazards Program (2017). The star shows our relocated hypocenter of the 2021 East Cape earthquake. The thin gray lines are the plate boundaries (Bird, 2003).
References


